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SURFACE-FRICTION COEFFICIENT IN TURBULENT FLOW AT A BOUNDARY LAYER

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Using Thompson two-parameter velocity profiles equations defining the dependence of the surface-friction coefficient on the integral characteristics of the boundary layer are obtained.

Because the initial system of differential equations is not closed, the calculation of a turbulent boundary layer requires the use of various kinds of empirical relations. In integral and quasiintegral methods of calculating turbulent friction, one of the closing equations is taken in the form of a dependence of the surface-friction coefficient on some parameters of the boundary layer. Relations of similar type are used in differential calculation methods assuming a polynomial specification of the frictional-stress distribution over the boundary-layer thickness. In most cases, it is borne in mind here that turbulent flow is described from the viewpoint of its local equilibrium, although calculation methods employing empirical data on the frictional drag and with the determination of a flow field with "inheritance" are known [1].

Numerous empirical dependences expressing the friction coefficient as a function of the Reynolds number referred to the longitudinal coordinate or the momentum-loss thickness are known. Single-parameter formulas of the form $c_f = f(Re)$, $c_f = f(Re_\theta)$ are valid at large Reynolds numbers for boundary layers of a plane plate; sometimes, it is used, together with the assumption that $H = \text{const}$, in calculations of gradient flows by an integral method. It is assumed here that the influence of the pressure gradient is taken into account intrinsically by the integral momentum relation.

Two- and three-parameter dependences are of greater accuracy, reflecting more completely the features of the flow in the boundary layer — in particular, with zero pressure gradient.

The most widespread approach is the semiempirical method based on the law of the wall, the formula of [2]

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$$c_f = 0.246 \cdot 10^{-0.678H} \mathrm{Re}_\theta^{-0.268}. \quad (1)$$

In a series of integral methods – see [3, 4], etc. – an equation derived from the Coles wake law [5] is used to determine the frictional stress at the wall

$$\frac{U_1}{v^*} = \frac{1}{k} \ln \frac{v_* \delta}{\nu} + \frac{2\Pi}{k} + B, \quad v_* = \sqrt{\frac{\tau_w}{\rho}} = \omega U_1, \quad \omega = \sqrt{\frac{c_f}{2}}. \quad (2)$$

Other equations of the friction law have been used. Some of these are noted in Table 1 [6-12]. In [1, 6, 8, 13], preference is given to the dependence $c_f = f(H, \mathrm{Re}_\theta)$ proposed in [14]. In [6], it was noted that the family of Thompson velocity profiles on which the dependence $c_f = f(H, \mathrm{Re}_\theta)$ is based is among the most perfect of these profiles. Numerous comparisons of calculated and experimental data have shown [1, 13, 15, 16] that two-parameter Thompson profiles approximate the velocity distribution in the boundary layer with high accuracy. This is also indicated by the analysis in [20] of the experiments of the Stanford conference [17] and the experimental investigations [18, 19]. The comparison in [20] of calculated velocity profiles with experimental data for 25 experiments, including 25 different flows, some of them relaxational, over a broad range of variation of H and Re_θ, invariably showed practically complete coincidence of the experimental and calculated velocity distributions.

Thompson represented the dependence of the surface-friction coefficient on H and Re_θ in the form of a network of curves $H = f(\mathrm{Re}_\theta)$ with the parameter c_f. This network was also given in [16]. In practical calculations, it is difficult to use a network of curves. Therefore,

TABLE 1. Surface-Friction Coefficients

№	Ref.	Friction law
1	[6]	$c_f = 0{,}0580 \left[\lg \frac{8{,}05}{H^{1{,}818}}\right]^{1{,}705} \mathrm{Re}_\theta^{-0{,}268}$
2	[7]	$c_f = 1{,}28 (\ln \mathrm{Re}_\theta)^{-1{,}74} \exp[-H(1{,}07+0{,}31 \ln(\ln \mathrm{Re}_\theta))]$
3	[8]	$c_f = 0{,}3 \exp(-1{,}33H) (\lg \mathrm{Re}_\theta)^{-(1{,}74+0{,}31H)}$
4	[9]	$c_f = 2\left[\frac{1}{0{,}41} \ln \frac{U_1 \delta^*}{\nu} + 2G - 4{,}25G^{0{,}5} + 2{,}12\right]^{-2}$, $G = \frac{H-1}{\omega H}$, $\omega = \sqrt{\frac{c_f}{2}}$
5	[7]	$c_f = 0{,}00810\left(\frac{100}{\mathrm{Re}_\theta}\right)^{(0.25-6{,}9\cdot10^4 k_l)}$, $k_l > 0$ $c_f = \frac{2}{(2{,}5 \ln \mathrm{Re}_\theta + 3{,}8)^2} - 0{,}00210k'\left(-\frac{k_l \mathrm{Re}_\theta^{1{,}163}}{0{,}0313}\right)^{0{,}668k_l}$, $k_l \leqslant 0$, $k' = (-k_l \cdot 10^7)^{0{,}125}$, $k_l = \frac{\nu}{U_1^2} \frac{dU_1}{dx} =$ $= -\beta \frac{c_f}{2\mathrm{Re}_\theta}$, $\beta = \frac{\theta}{\tau_w} \frac{dp}{dx}$
6	[10]	$c_f = 2\left[2{,}44 \ln \Gamma - \frac{15}{\sqrt{\Gamma}} - \frac{6}{\Gamma} + 10\left(3 + \frac{0{,}14}{\omega^2} G_0^2\right)^{1/4}\right]^{-2}$, $\Gamma = 1{,}2\,\mathrm{Re}\left(\frac{10}{\omega} + \frac{G_0}{\omega^3}\right)^{-1}$, $G_0 = -\frac{\delta}{U_1} \frac{dU_1}{dx}$
7	[11]	$\frac{c_f}{c_{f_0}} = 1 + 0{,}1367f + 0{,}015f^2 + 0{,}00337f^3$, $f = \frac{\theta^2}{\nu} \times$ $\times \frac{dU_1}{dx}$, c_{f_0} is the drag coefficient when f = 0
8	[12]	$\frac{c_f}{2} = \left(\frac{0{,}000565}{0{,}005934 - \Gamma} - 0{,}00291\right) \mathrm{Re}_\theta^{-0{,}25}$, $\Gamma = \left(-\frac{\theta}{\rho U_1^2} \frac{dp}{dx}\right) \mathrm{Re}_\theta^{0{,}25}$

in [1], for example, a preliminary tabulation of the functions $c_f(H, Re_\theta)$ and $Re_\delta(H, Re_\theta)$ was recommended. The problem may evidently be significantly simplified if an analytical description of the network of curves or its approximation is obtained.

The Thompson profile is used here

$$\frac{U}{U_1} = \gamma \left(\frac{U}{U_1}\right)_{in} + 1 - \gamma. \tag{3}$$

The weighting function γ is equal to unity in the range $0 < y/\delta \leqslant 0.05$ and to zero close to the external edge of the boundary layer, where $0.95 < y/\delta \leqslant 1$. Thompson represented the dependence $\gamma(y/\delta)$ by a curve generalizing the experimental data. In [13], the interval $0.05 < y/\delta \leqslant 0.95$ was divided into three sections and was approximated within each section by a second-order polynomial.

It may be shown [21] that the function γ is related to the Coles wake function W by a simple linear dependence $\gamma = 1 - 0.5W$. If the Hinze approximation for W is adopted ($W = 1 - \cos \pi y/\delta$), then the function γ will be described by the following equation in the range $y/\delta = 0.05$-0.95

$$\gamma = 0.5 \left(1 + \cos \frac{10}{9} \pi\eta\right), \quad \eta = \frac{y}{\delta} - 0.05. \tag{4}$$

The difference in the values of γ calculated from Eq. (4) and the Galbraith and Head dependence is slight [21].

Thus, the expression obtained for the Thompson velocity profile is

$$\frac{U}{U_1} = 0{,}5\left[\left(1 + \cos\frac{10}{9}\pi\eta\right)\left(\frac{\omega}{k}\ln \mathrm{Re}_y\omega + B\omega\right)\left(1 - \cos\frac{10}{9}\pi\eta\right)\right], \tag{5}$$

$$\mathrm{Re}_y = \frac{U_1 y}{\nu}.$$

The parameters k and B in the equation are taken to be 0.4186 and 5.45, according to the data of Patel.

In the laminar sublayer and the buffer layer, the velocity distribution is taken in the form [13]: when $0 < y^+ < 4$

$$U^+ = y^+, \tag{6}$$

when $4 < y^+ < 30$

$$U^+ = 4{,}187 - 5{,}745 \ln y^+ + 5{,}11 (\ln y^+)^2 - 0{,}767 (\ln y^+)^3. \tag{7}$$

The distribution in Eq. (7) was proposed by Dvorok.

In the range $30\nu/\omega U_1 \delta < y/\delta < 0.05$, where $\gamma = 1$, the velocity profile is described by a logarithmic wall law

$$\frac{U}{U_1} = \omega\left(\frac{1}{k}\ln \mathrm{Re}_y\omega + B\right). \tag{8}$$

Substituting Eqs. (5)-(8) into the expressions for the displacement thickness and the momentum-loss thickness gives

$$\frac{\delta^*}{\delta} = 0{,}5 - \omega(0{.}80095 + 1{,}1943 \ln \mathrm{Re}_\delta\omega) + \frac{50{,}7}{\mathrm{Re}_\delta}, \tag{9}$$

$$\frac{\theta}{\delta} = 0{.}1125 + \omega(0{.}65687 \ln \mathrm{Re}_\delta\omega - 0{,}019389) - \omega^2\left[2{,}21090(\ln \mathrm{Re}_\delta\omega)^2 + 1{.}8667 \ln \mathrm{Re}_\delta\omega + 2{,}94987\right] - \frac{60{,}48}{\mathrm{Re}_\delta}(1 - 16{.}44\omega). \tag{10}$$

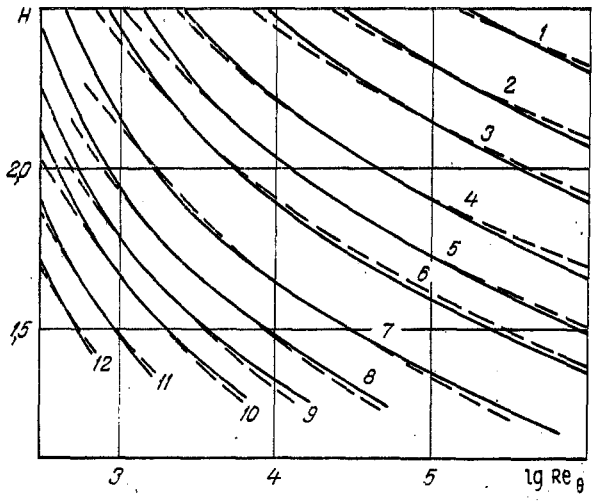

Fig. 1. The dependence $c_f = f(Re_\theta, H)$ according to Eqs. (9) and (10) (continuous curves) and Eq. (11) (dashed cuves): 1) c_f = 0.0002; 2) 0.0003; 3) 0.0004; 4) 0.0006; 5) 0.0008; 6) 0.001; 7) 0.0015; 8) 0.002; 9) 0.0025; 10) 0.003; 11) 0.004; 12) 0.005.

The equations obtained give an analytical description of the network of Thompson curves (Fig. 1). When ω = const, specifying different values of Re_δ leads to results for δ^*/δ and hence for $H = \delta^*/\theta$ and $Re_\theta = Re_\delta\theta/\delta$. Finally, the required dependence $c_f = f(H, Re_\theta)$ is obtained. In constructing the network of curves in [14], the values H and Re_θ were obtained by graphical interpolation.

Equations (9) and (10), determining the surface-friction coefficient in implicit form, also include other characteristics, which are usually determined in the course of boundary-layer calculation. In contrast to the above empirical dependences, Eqs. (9) and (10) allow $c_f = f(H, Re_\theta)$ to be calculated over a broad range of variation of the arguments.

It is known that the formula of [2], which is most often used, is obtained on the basis of experimental data bounded by the intervals $1.2 \leqslant H < 2.0$, $10^3 < Re_\theta < 2 \cdot 10^4$. However, beyond the limits of this region, as noted in [22], the error may reach 40%.

As is evident from Fig. 2, the curves of $H = f(Re_\theta)$ with c_f = const plotted from the formulas of [6-8] also lie close to the Thompson curves only at values of H and Re_θ in the middle of the given range of variation. When $c_f > 0.002$ and $c_f < 0.008$, the deviation becomes considerable. The explanation for this is that the Thompson model is based on a definite mathematical model, whereas the remaining formulas directly approximate experimental data in a limited range of variation of the independent variables.

The agreement with the Thompson curves may be improved by some complication of the approximating expressions. The dashed parametric curves in Fig. 1 are plotted according to the formula

$$c_f = 0{,}000423 \exp\left[-KH + 14{,}497\,(\ln \mathrm{Re}_\theta)^{-0{,}6}\right], \qquad (11)$$

$$K = 1{,}543 \text{ when } c_f \geqslant 0{,}001,\ K = 1{,}299 c_f^{-0{,}025} \text{ when } c_f < 0{,}001,$$

taking account of the nonlinearity of the dependence $H = f(Re_\theta)$. Agreement of the continuous and dashed curves is observed over a sufficiently broad range of Re_θ. Equation (11) is simpler than the initial Eqs. (9) and (10) but, of course, lacks something in accuracy.

Equations (9) and (10), which describe the law of friction in a turbulent boundary layer, also allow the boundary-layer thickness δ to be determined using the integral characteristics δ^* and θ.

Close to the external edge of the boundary layer, even a small error of the velocity profile leads to marked change in the boundary-layer thickness δ, equal to the coordinate at which $U = 0.995U_1$. Therefore, the determination of δ directly from the measured or calcu-

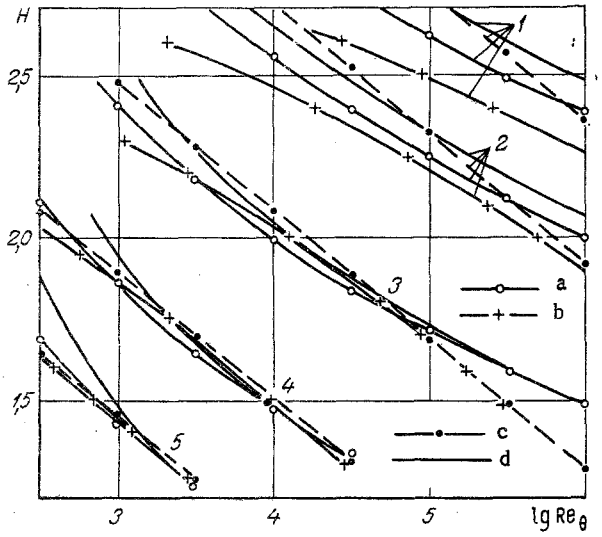

Fig. 2. Comparison of the curves of $H = f(Re_\theta)$ at $c_f = const$ according to the formulas of [8] (a), [6] (b), and [7] (c); continuous curve (d) correspond to the Thompson model: 1) $c_f = 0.00015$; 2) 0.0003; 3) 0.0008; 4) 0.002; 5) 0.004.

lated velocity profile is unreliable. At the same time, the integral characteristics of the boundary layer δ^* and θ are calculated, as is known, with sufficiently high accuracy. This accuracy may also be transferred to the determination of the boundary-layer thickness if Eq. (9) or (10) is used. The value of δ is found here by the method of successive approximation.

NOTATION

c_f, friction coefficient; $H = \delta^*/\theta$, form parameter; δ^*, displacement thickness; θ, momentum-loss thickness; U, current velocity over the boundary layer thickness; U_1, velocity at the external boundary of the boundary layer; $(U/U_1)_{in}$, velocity distribution according to the wall law, Eq. (8); δ, boundary-layer thickness; $v_* = \sqrt{\tau_w/\rho}$, dynamic velocity; $U^+ = U/v_*$; $y_+ = y/v^*$; Π, parameter in the Coles wake law; $Re_y = U_1 y/\nu$; $Re_\delta = U_1\delta/\nu$; $Re_\theta = U_1\theta/\nu$.

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MATHEMATICAL MODEL OF HYDRATE FORMATION IN THE FLOW OF MOIST GAS IN TUBES

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The problem of hydrate formation in the flow of moist gas in tubes is formulated, under the assumption that the temperature of hydrate formation depends not only on the pressure but also on the water-vapor concentration at the phase-transition surface.

The problem of hydrate formation in gas pipelines was first considered in [1-3], where the conditions of hydrate formation were described, and recommendations for the prevention of hydrate formation in gas-pipeline operations were made; these reduce to the need to dry the gas, remove condensed water, and employ pipeline operating conditions that eliminate the possibility of hydrate formation. The problem was then discussed in [4-10], where attempts were made to determine in advance the sites of possible obstruction of the pipeline by hydrates, and to give a quantitative calculation of the mass of hydrate forming in the course of gas transport. However, these works have a series of deficiencies. In [8], for example, the mass rate of hydrate formation was estimated, but no mention was made of which section of the pipeline was subject to hydrate deposition. In [9], the region of possible hydrate formation was determined on the basis of the thermodynamic conditions of moisture removal from the gas, but the process of hydrate deposition itself was not considered. In [10], the model of hydrate formation was constructed from the numerical solution of the equations of nonisothermal motion of a real gas, and the action of the hydrate obstruction was modeled by a local resistance with an unknown drag coefficient, which is a significant deficiency of the model.

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